An Application of Weibull Analysis to Determine Failure Rates in Automotive Components

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1. Abstract

This paper focuses on the automotive component failure rate detection using Weibull analysis and other survival analysis techniques. Detailed attention is paid to three areas: 1) overall failure rates are described statistically first, and data cleaning and definitions of 'failed' and 'censored' data within the research time or warranty period are made; 2) Kaplan-Meier life curves and Log-rank tests are used to compare the component reliability over time and explore risks factor related to the component failure; 3) Weibull regressions, with two and three parameters, are applied to fit real-world reliability data from different test conditions, and to predict the automotive component failure trend over future time. The analysis results agree well with real-world test data, and provide reasonable prediction of future failure trends.

2. Introduction

Automotive components fail over time, especially beyond the warranty time period. It is important to study the reasons why automotive components fail, and to predict the component reliability trends, associated with various manufacturing, environmental and testing conditions.

For effective use of a predictive failure analysis of automotive components, all of the failures within a specific driving time period need to be captured. Because any repairs to failed components are free to the owner and paid by the manufacturer during the manufacturer's *basic warranty* coverage period (usually 36 months or 36,000 miles), most repair records are kept well. These are available to investigators and provide information on what components have been repaired or replaced at specific mileages and time in service. However, as soon as the basic warranty (free repair) period is over, and owners are required to pay for the repairs, there is no central repository to collect comprehensive information on failures due to data availability, which may lead to limited or incomplete analysis beyond the relatively short basic warranty window. In order to overcome this obstacle and to capture more insightful failure information within a relatively longer period, a data analysis technique was developed to utilize the manufacturer's *extended warranty* data to create a richer and more representative sample, from which the data analysis could be efficiently utilized, although it is not easy to completely avoid data bias when the extended warranty programs are slected. Most manufacturers will offer an extended service plan to an owner that provides coverage for a specified period beyond the basic warranty. For example the *extended warranty plan* may be for 5 years from purchase and 60,000 miles, or 5 years/75,000 miles, or 5 years/100,000 miles, or for 4 years/75,000 miles, etc. and that there are typically extended service plans offering different levels of coverage on different components (e.g., Gold Plan, Platinum Plan, Powertrain Plan). The driver may choose a plan that best fits his/her need. The data technique developed here requires that 1) all vehicles sold with all extended service plans be identified, 2) the plan(s) that cover repairs on the subject component be identified, 3) that a calculation is made to determine average miles per month driven for the subject vehicles using records of warranty repairs, which provide time in service and mileage at the time of repair. All detailed records of repairs associated with time and mileages are then used for data analysis.

For example, one powertrain component under consideration has a population sample size of over 200,000, the component failures starts around 30,000 miles, with extended service plans for 60 months, or 100,000 miles (warranty period or agreements), whichever comes first, the data set is termed as '*Old K60'* data of type-1 in this study. Second type of test data of this same component may have a different failure mode or with a mix of failure modes. For this type-2 test, there are three new data sets of interest – first new data set has the capture information of 60 months and 100,000 miles, i.e., the maximum month to failure is 60 months, and maximum mileage of 100,000 miles ('*New K60' data*); Similarly, 2nd new data set has the information of 48 month/100,000 mile capture, i.e., the maximum month to failure is 48, and with maximum mileage of 100,000 ('*New K48*'); The third contains the info of 72 month/100,000 mile capture ('*New K72'*). In total, there are four sets of data for type 1 and type 2 tests of this same component, which are summarized in Table 1.

Most reliability test data are closely time-related, and failures happened within the warranty time window or beyond, a technique of 'time-to-event' or survival analysis is very suitable for such reliability data, especially Weibull model is a well established tool to fit the test data and to predict the future failure trend beyond the available test duration. The main goals of this project are to apply a Weibull model to the automotive component reliability analysis, and then explore the failure rate over service time or mileage, and to provide some statistically-based insights into the component failures by the following process:

- Provide descriptive summary of available failure data of a component;
- Verify the reliability difference over time of the component under two different conditions;
- Explore the component failure probability over test time, and compare the failure rates of the same component from a few different data sets;
- Improve the data fitting and prediction by using a three-parameter Weibull model, and compare with real-world test results.

3. Descriptive Summary of Data Sample

The focused automotive component test data, with service plan for 60 months, or 100,000 miles ('Old K60' of Type-1), can be divided into two sub-groups: The first group is "failed event" data (234 failures as following Table 1); The second group, 'Suspension' or 'Censored' group, has 4483 observations and has no failures within 60 months, further, their future failure behavior beyond 60 months are unknown. The service time of the 'censored' group, at least, all passed the 'Cut-Off' line of 60 months. Figure 1 on next page is helpful to explain the failures within and beyond a warranty time, or research time window.

The similar descriptive summaries of three new data sets (New K60, New K48, and New K72) are also listed in following Table 1, with the failure data descriptive summary.

Table1: Descriptive Statistics for Failed Components

	mean	**Std Dev.**	**Min**	**Max**	**failure/ all %**
Type-1: **Old K6**0 Data (234 failures, 4483 censors)					
Month	44.3	9.92	19.0	60	5.0%
Type-2: **New K60** Data (280 failures & 4474 censors)					
Month	43.3	10.14	14.0	60	5.89%
Mile	70,597	17,326	27,920	126,288	
Type-2: **New K72** Data (164 failures & 2282 censors)					
Month	42.24	13.68	9.0	68	6.70%
Mile	60,097	17,513	21,290	98,958	
Type-2: **New K48** Data (54 failures and 871 censors)					
Month	32.3	10.3	10.0	48.0	5.80%
Mile	69,798	20,370	28,888	97,687	

The ratio of 'failures /all %' (all=failures + censors)' in the last column in Table 1 is an important parameter that gives the overall failure rate at the end of service time, and will be used later to compare with the failure rate predicted by a Weibull model at the same time. For example, the failure rate at the time of 60 months is 5%, for data 'Old K60', or 5.89% for 'New K60' data.

4. Methods of Modeling Survival and Failure Rates

It is of great interest to observe the component failure rate, F(t), or from an opposite point of view, the survival probability varying over a test time, S(t), while there is a simply relationship between the two: '$S(t)= 1 - F(t)$', i.e., 20% failures means 80% survival rate among a fixed sample.

One of the most useful tools to compare the survival probability over time is a method proposed by

Kaplan and Meier [1]. The Kaplan-Meier survival curve is described by the following formula:

$$\hat{S}(t) = \prod_{t_i <= t} \left(1 - \frac{d_i}{n_i}\right) = \prod_{t_i <= t} \left(\frac{s_i}{n_i}\right) \qquad (1)$$

Where 'd_i' is 'deceased' subject or failed automotive component, and 's_i' is the 'survivor' or 'alive' component, and 'n_i' is the total (both failed and suspension components) in the study at any moment beyond time zero.

Or, turning the problem around, the failure probability over test time, F(t), equal to '1-S(t)', it can be further expressed by following Eq. (2) in Weibull model [2]:

$$F(t) = 1 - e^{-(t/\eta)^{\beta}} \qquad (2)$$

Or, equivalently it can be visualized by the following *'linear'* transformation, as Eq. (3): [2]

$$\log(-\log(S(t)) = \beta \log(t) - \beta \log(\eta) \qquad (3)$$

In the above Eq.(3), S(t) is survival function, which can be estimated from the Kaplan-Meier curve discussed earlier, and F(t) of Eq. (2) is the accumulation of failure probability as time increases, here '**β**' is regarded as the 'Slope' of the 'linear' plot, or 'Shape' parameter, and '**η**' is a 'Scale' parameter and is related to the intercept of the 'linear' plot.

When a plot of test data is not visualized as a *'linear'* plot as Eq. (3), especially at the earlier time stage, a Weibuull with three parameters, as Eq. (4), provides a better data fitting, where a time shift, or threshold, t_0, is included as Eq. (4) -

$$F(t) = 1 - e^{-((t - t_0)/\eta)^{\beta}} \qquad (4)$$

Three cases are studies in details using above Eq. (1) to (4), from some investigation data and examples, as shown as following Sections 5, where Case1 compares the reliability curves over time of two different conditions; and Cases 2-3 applies Weibull models, with two or three parameters, for data fitting and future trend predictions.

Computing procedures by SAS Institute, *'LifeTest', 'LifeReg', and 'Reliability*', are used for calculations [3]. Some extra attentions should be paid to - how to clean the time data using SAS program; to determine the test interval from start to end; to convert the time data from calendar time (with different starting time each) to study time where all subjects have the same starting time; and to determine the relative status of 'censor' over time. [1,3]

5. Case Studies of Modeling Survival and Failure Rates

***Case 1**: Compare Survival Rates Over Time of Two different Conditions*

It is often required to investigate automotive component reliability or survival rates over time under two different conditions, either manufactured during different time periods, or, by two different designs, or being used under different environmental conditions.

Here is one example of discussing 'failed event' and 'censor' within the research time window and beyond – there are 234 failures from data set of 'Old K60' within the study time of 60 months, however, the 'Suspension' or 'Censored' 4483 observations have service time beyond the 'Cut-Off' line of 60 months, and their future statuses beyond 60 months are unknown (see Figure 1).

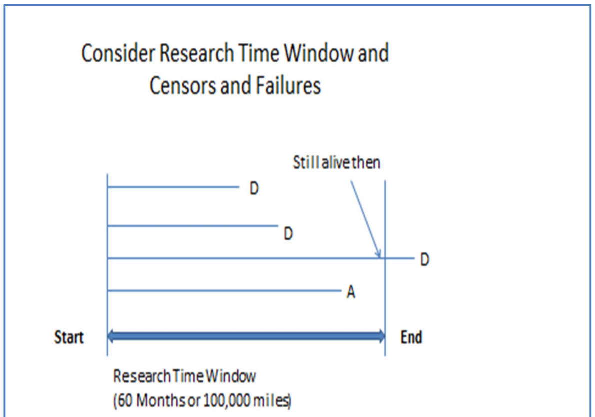

Figure 1: **Research Time Window, censor & failures**

In this project, the research time frame is '0-60' months (warranty duration or other agreement), some components have test time less than 60 months without failures, and they are treated as 'censored' data; all 'suspension' data are still 'alive' without failures beyond 60 months ('Censors', or '0'), and their service time are all assumed to be at least 60 months. On the other hand, all 'failed' data (234 failures, coded as '1') have specific time of failures for each, prior to 60 months (Figure 1).

The following Kaplan-Meier life curves (Figure 2), using 'Old K60' failed data only, are studied in detail to verify if the reliability of products built during 'Period A' might be different from the reliability of products built at other time.

Figure 2 indicates that no significant difference of reliability is observed between the products built during 'Period-A' (orange color) and other dates ('blue' or '0'). The log-rank test, which compares two survival rate curves over time, provides a p-value of 21% for the two Kaplan-Meier plots of Figure 2, where 'X-axis' has a unit of 'month' and 'Y-axis' displays the probability from zero to 1.0.

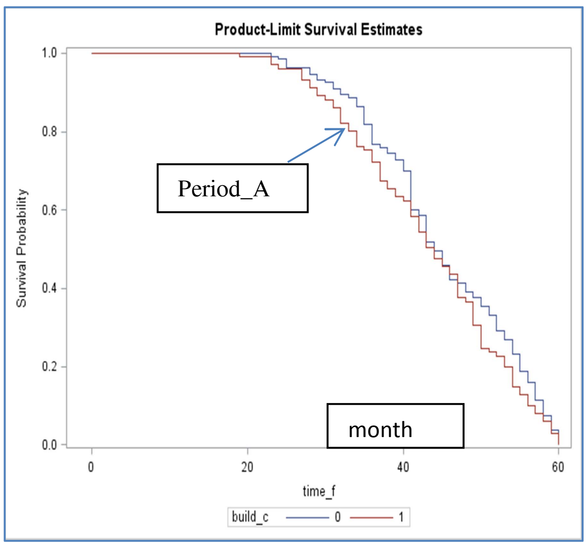

Figure 2: Survival Plots of Component built during 'Period_A' (Orange Color) vs. Other Dates.

On the other hand, a significant difference (p-value <0.0001) of reliability is observed (Figure 3), from the failed data of another automotive component whose mileage (X-axis) information for each is available, under two different using conditions (1673 failures under Condition-A, and 531failures under Condition-B).

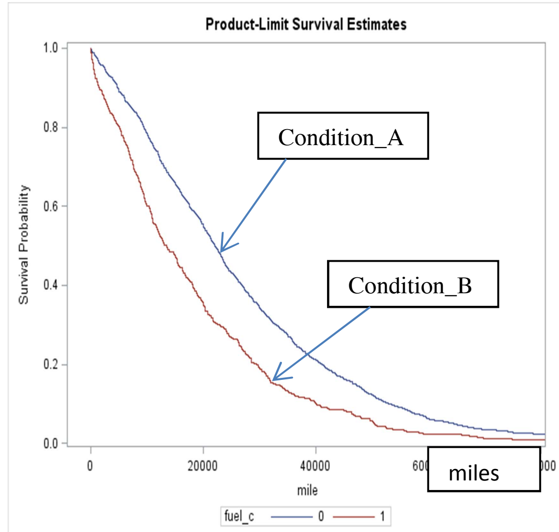

Figure 3: Survival Plots of Two Groups with two Using Conditions, p-value<0.0001

One useful feature of the Kaplan-Meier life curve is to permit comparison of the survival rates over time between two different conditions, and evaluate the effect of one single factor (such as manufacturing time as Figure 2, or using condition as Figure 3) on reliability, if more risk factors are considered simultaneously, the Cox proportional hazard model is a better tool to evaluate multiple risk factors [3].

__Case 2__: Weibull Modeling with 'Linear' Plot using Different Data Sets

One specific automotive component can be tested under different conditions, and the different tests (such as load or temperature) may lead to different failure modes. A 'linear' Weibull model, based on Eq. (2) –(3), is used first to plot the failure rates over time. For various tests of this same component, the similar approach of treating 'failed' events ('1') and 'censor' data ('0'), as Figure 1 of Case 1, is applied to all data sets: 'Old K60', 'New K60', 'New K72' and 'New K48'.

Figure 4 shows the 'linear' plots of Weibull modeling of failure probability from data set of 'Old K60', where 'X-axis' has a unit of 'month' and 'Y-axis' displays the probability from zero to 1.0.

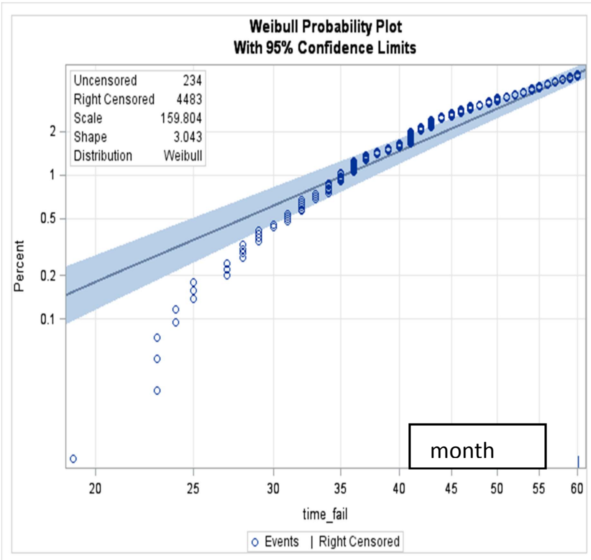

Figure 4: Weibull Plot with Data of 'Old K60', where 234 are failures and 4483 are "suspension"

In Figure 4, the failure rate will reach approximately 5 percent afters 60 months, and this prediction agrees very well with the test result as Table 1 (the last column and 3rd row). On the other hand, the 'linear' Weibull model with only two parameters of '**β**' ('Slope' or 'Shape') and '**η**' ('Scale') does not fit the earlier time data very well (under 30 months), more discussions will be given later in 'Case 3', where the 'non-linear' plot under 30 months will be described further.

The data fitting results from using all data ('failed' and 'censored' data) and from 'failed' data only are listed in Table 2.

Table 2: Weibull Parameters and Failure Prediction (Two Samples out of 'Old K60' Data)

Parameter	'Failed, (1)' and 'Suspension, (0)'	Failed data ('1') Only
β (slope/shape)	3.04	5.24
η (scale)	159.8	48.2
5% failure	60 months (*)	23 months

When a few test data sets of the same component are available, it is of interest to compare the failure rates from using old data (Old K60), together with the results using new data (New K60, New 72K, and New 48K), as shown in Table 3. The failure rate from the shortest warranty program (48 months) is slightly faster, which reflects the reality of components in use.

Table 3: Weibull Parameters and Failure Prediction (One Old & Three New Data Samples)

parameter	Old 60K	New 60K	New 72K	New 48K
β (shape)	3.04	2.84	1.79	2.20
η (scale)	159.8	161.1	311.9	172.4
5% fail.	60 mo	56 Mo	59 Mo	45 mo
10% fail	76 mo	73 mo	88 mo	62 mo

Figure 5 provides the Weibull modeling results using two data sets of 'Old K60' and 'New K60' (with the same warranty program) as one example. The predictions from the two different data sets agree well, and indicates that the same component fails at a similar rate although under two slightly different test modes and conditions.

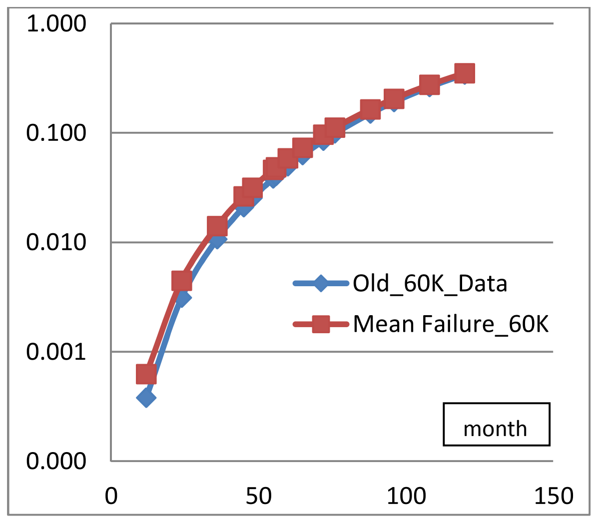

Figure 5: **Compare Mean Failures (Y-axis) vs. Month (X-axis) from Old Data (Old K60) vs. New Data (New K60).**

<u>Case 3</u>: *Improving Data Fitting and Prediction by Considering the Time Shift*

The Weibull 'linear' plot with only two parameters, as Figure 4, is not fitting data well enough, especially at the earlier time stage (time <30 months). A Weibull model with three parameters, shown as Eq. (4), provides a better fit. The data fitting parameters displayed in Figure 6 are for the'Old K60' data.

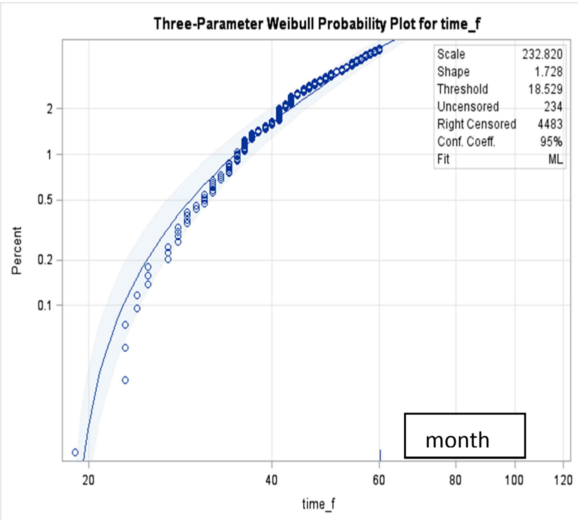

Figure 6: **Failure (Y-axis) vs. Month (X-axis) using Eq (4), 'Old 60K' data**

Furthermore, the three parameters of Figure 6 (Slope β=1.728, Scale η=232.82, and threshold or time-shift t_0=18.529) are used in Eq. (4), and provides the future trend prediction as Figure 7.

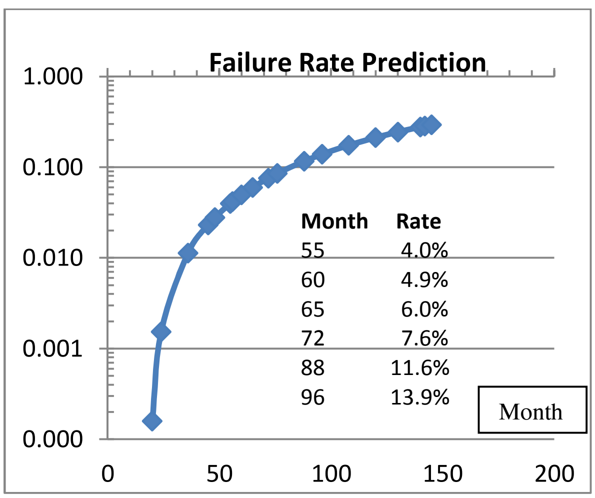

Figure 7: **Mean Failure Prediction vs. Month using Results of Figure 6, by Eq(4), with Slope (1.728), Scale (232.82), and time-shift (18.529), 'Old 60K' data.**

A similar Weibull curve as Figure 6 but using 'mileage' for the 'X-axis' is also plotted from a data set 'New K60', and the plot of failure against mileage can be more intuitive and realistic sometimes than the similar curve of failure against time (a car can be kept in a garage for a long time).

6. Conclusion

- Modeling of automotive component reliability is a data mining and learning process at NHTSA. From the simple statistical description, to estimation of a reliability curve over time, to a proper mathematical model to fit the test data and to predict the future component failure trend.

- Employing the Kaplan-Meier life curve permits us to compare the component reliability over time between two different conditions, and to evaluate the effect of one single factor with statistical reliability.

- A Weibull model with two parameters (slope, β, and scale, η) can reasonably predict the mean failure with a 'linear' model, while a Weibull model with three parameters can treat some 'nonlinearity' at earlier time stage much better. For example, about 5 percent of products fail when service time has reached 60 months, and this prediction agrees well with the known test.

- The modeling results from using a few different test data sets from a same component provide meaningful comparisons, and such comparisons permit insights into component failure modes under different manufacturing regimes.

7. References

1. Kaplan, E. L. and Meier, P. (1958), "Nonparametric Estimation from Incomplete Observations", J. of the American Statistical Association, 261, 719-724.
2. Abernethy, Robert, *The New Weibull Handbook*, (3rd Edition), 1999
3. Allison, P. (1999), *Survival Analysis using SAS: A Practical Guide*, SAS Publishing.